Boban 12/4

II

# The Method of Side Conditions

Ben De Bondt + Obrad Kasum

$\phi$ infinitary prop formula in CNF

A solution is $\mu : V \to \{0,1\}$ st $\mu \models \phi$.

- Can we add a solution by forcing?

$\underline{\mathbb{H}}(\phi)$ canonical forcing for this

| | | | |
|---|---|---|---|
| I | $c_0 \in \phi$ | $c_1 \in \phi$ | ... |
| II | $l_0 \in c_0$ | $l_1 \in C_1$ | ... |

$\omega$ moves

II must not pick $l_i, \neg l_i$. for any $i$.

$G(\phi, p)$, $p$ a partial assignment.

Closed game for player II, hence determined.

$p \in \mathbb{H}(\phi)$ if $\mathrm{II} \uparrow G(\phi, p)$.

$q \leq p$ if $q \supseteq p$.

- $\mathbb{H}(\phi)$ may collapse cardinals etc

Can we add an assignment w/o causing chaos, we beg a

point $\mathbb{P} \in \mathcal{K}$, $\mathcal{K}$ a nice class of forcings eg semiproper

$\mathbb{P}_0 = \mathbb{H}(\phi)$

$\mathbb{P}_1$ : Fix $\theta$, $M \prec H_\theta$ ctbl, $p \in \mathbb{H}(\phi)$

| | | |
|---|---|---|
| $G(\phi, M, p)$ | I | $C \in \phi$ |
| | II | $l \in C$ |

$p_{n+1} = p_n \cup \{\phi\}$

$P_0 = p$

$\vee\!\!/$

$P_1$

$\vee\!\!/$

$P_2$

$\vdots$

$P_{n-1}$

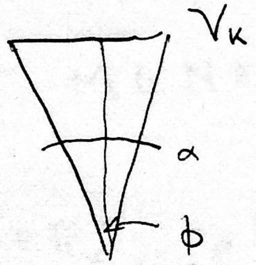
Player I can play a dense $D \subseteq \mathbb{P}_0$, $D \in M$

Player II picks $a \in D$, $p_n \le p_{n-1}$, $a$

$$Hull(M, \{a\}) \cap \omega_1 = M \cap \omega_1 .$$

M is a good model if II has a winning strategy ~~for~~ for $G(\Phi, M, p)$ $\forall p \in M \cap \mathbb{P}_0$.

- $p \in \mathbb{P}_1$ : $p = (\mathcal{M}_p, w_p)$ $|\mathcal{M}_p| \le 1$.
- M good model, $w_p$ II $\uparrow$ $G(\Phi, M, w_p)$.
- We want this to "converge"

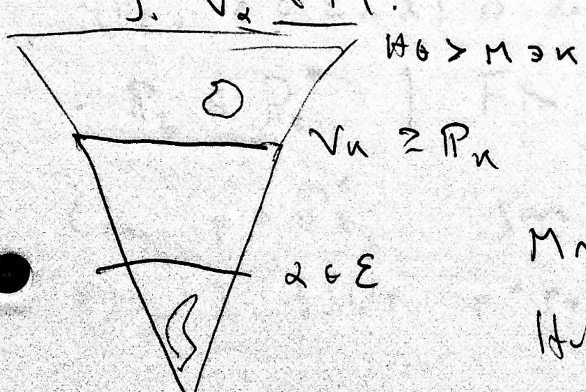



$\mathcal{E} = \{\alpha : V_\alpha \prec V_\kappa\}$

$V_\alpha \prec V_\kappa$ $\alpha \in \mathcal{E}$, $\alpha$-models

M is an $\alpha$-model if

1. $M \prec \hat{M} \models ZFC^-$ where $\hat{M}$ = transitive closure not ctbl
2. $\hat{M} = Hull(M, V_\alpha) = \{f(\bar{x}) : f \in M, \bar{x} \in V_\alpha\}$
3. $V_\alpha \in M$.




$M \cap \kappa \subseteq \alpha$.

$Hull(M, V_\alpha)$

$\mathrm{Hull}(M, V_\alpha) \cap \kappa = \alpha$

$\pi$ trc

$\overline{\mathrm{Hull}(M, V_\alpha)}$ $\quad \pi[M]$ is an $\alpha$-model

$\shortparallel$

$M \restriction \alpha$

$p \in M$

$\shortparallel$

$(M_p, w_p)$

$C_\alpha = \{M : M \ \alpha\text{-model}\}$

$C = \bigcup C_\alpha$

Preconditions

$p \in \mathbb{P}_\kappa^*$ if $p = (M_p, w_p)$

- $w_p \in \mathbb{H}(\phi)$
- $M_p$ is a finite $\in$-chain of models in $\mathcal{C}$.

$M \in N \in M_p \to \lambda_M < \lambda_N$.

"the $\kappa$ of the model"

$p \in \mathbb{P}_\kappa^*$, $p \restriction \lambda = (\{M \in M_p : \lambda_M < \lambda\}, w_p)$

$q \leq p$ iff $w_q \supseteq w_p$ & $\forall M \in M_p \ \exists N \in M_q \ \lambda_M = \lambda_N$

and $M \leq_{\omega_1} N$. (cannot increase $\omega_1$)

$\delta_M = M \cap \omega_1$.

We define $\mathbb{P}_\lambda$, $\lambda \in \mathcal{E}$ by induction.

- $(\mathbb{P}_\xi, \ \xi \in \mathcal{E} \cap \lambda)$ is defined
- $\mathbb{P}_\xi \subseteq \mathbb{P}_\xi^*$ | Fix $\lambda \in \mathcal{E}$

Given $p \in \mathbb{P}_\lambda^*$, game $G_\lambda(p)$

Player II builds $p = p_0 \geq p_1 \geq p_2 \cdots$ length $\omega$

I . . . $Q_n$ ✓ questions

II . . .

I $Q_n$: $C_n \in \phi$

II $l_n \in C_n$ $\quad p_n = (\mathcal{M}_{p_n}, w_{p_n} \cup \{l_n\})$

I $Q_n$: $(M, D)$, $\overset{\omega}{M} \in \mathcal{M}_{p_n}$, $D \subset M$ dense in $\mathbb{P}_{\lambda_n}$

II picks $q \in D$, $p_{n+1} \le p_n, q$

$$\underset{\mathbb{P}^*_\lambda}{\in}$$

$$\mathrm{Hull}(M, \{q\}) \cap \omega_1 = M \cap \omega_1$$

$$(\delta_{\mathrm{Hull}(M,\{q\})} = \delta_M)$$

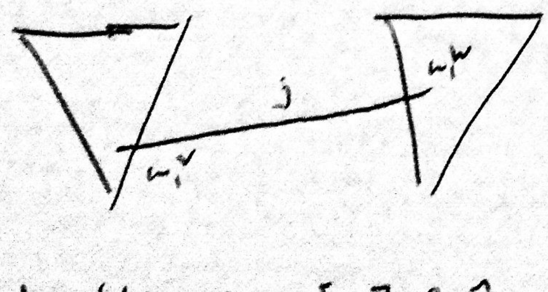
$p \in \mathbb{P}_\lambda$ iff II wins $G_\lambda(p)$.

(Kasum) AS-consistency.

$\phi$ is AS-consistent if for every st. pt.

In $V^{\mathrm{Col}(\omega_1,\kappa)}$ $\forall \mu \in V^{\mathrm{Col}(\omega_1,\kappa)}$ $\mu \models \phi$ $\exists j : V \to W$

crit $j = \omega_1^V \in j(S)$




$\exists \hat{\mu} \models j(\phi)$ st $j[\mu] \subseteq \hat{\mu}$.